ON THE THEORY OF 'Q'-METER AND ITS CORRECTIONS*

By V. V. L. RAO

(*Received for publication, February 13, 1942*)

INTRODUCTION

In communication engineering, 'Q,' the ratio of effective reactance to resistance at any frequency has great significance as a figure of merit of a coil or a condenser.

$$Q \text{ for a coil is denoted by } Q_L = \frac{\omega L_e}{R_e}$$

and $$Q \text{ for a condenser is denoted by } Q_C = \frac{1}{\omega C_e R_e}$$

where L_e, C_e, R_e are the effective values at the frequency $f(\omega = 2\pi f)$ considered.

In order to evaluate the 'Q' of a coil at different high frequencies, one has to measure carefully both L_e and R_e at each one of the frequencies concerned and then calculate the corresponding values of Q. To obviate this laborious process a few British and American firms have recently marketed a very ingenious and versatile meter called the circuit magnification or 'Q' meter, on which the values of Q between 10 and 500, and even up to 625, can be read off the meter directly in the frequency range 50 *kc/s* to 50 or even 75 *mc/s*. The accuracy of measurement of Q value in the Marconi Ekco 'Q' meter, Type TF 329D, is $\pm 5\% \pm 5$ up to 10 *mc/s*. Its frequency range is 50 *kc/s* to 50 *mc/s*. The versatility of the meter will be seen from the following list of measurements that can be made with such a meter :

(1) Q of coils and condensers
(2) Self-capacity of coils
(3) Inductance of coils
(4) Capacity of condensers
(5) Power-factor of condensers
(6) Dielectrics and high resistances
(7) Low Impedances

* Communicated by Prof. P. N. Ghosh.

(8) Transmission-line Constants—

(a) Characteristic Impedance

(b) Attenuation Constant

(c) Wave velocity.

THEORY

A simple theory of this meter is worked out below.

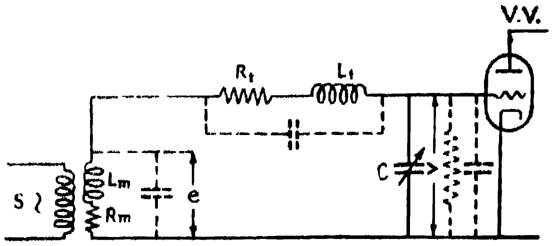

FIG. 1
Simplified circuit of a Q-meter

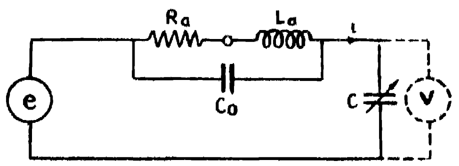

FIG. 2
Schematic-equivalent circuit of a Q-meter

Fig. 1 shows the simplified circuit of a Q-meter and Fig. 2 its equivalent circuit. S is an oscillator covering the desired frequency range and injecting a constant voltage 'e' into the tuned circuit by means of a small coil of inductance 'L_m' and resistance 'R_m' ($L_m = .05 \mu h$ and $R_m = 0.04 \Omega$ in the Marconi Ekco meter). V.V. is a valve-voltmeter, measuring the voltage across the condenser for convenience. Actually, at resonance, the voltage across the coil and that across the condenser are equal in magnitude. R_a and L_a represent the apparent values of resistance and inductance respectively, and C_0, the self-capacity of the coil, at the frequency in question. R_a and L_a are lumped so as to include R_m, the internal resistance, and L_m, the internal inductance of the source.

Then, the apparent Q of the coil is given approximately by the ratio $= V/e$, when the coil is tuned to resonance by the variable condenser C. When e is a known fixed value (say, 20 mv) the valve-voltmeter can be calibrated to read 'Q' directly as shown below :—

Let i be the current in the series circuit (*neglecting* C_0, the self-capacity).

Then
$$i = \frac{e}{R_a + j\omega L_a + \frac{1}{j\omega C}}$$

If $\quad Z_a = R_a + j\omega L_a, \quad i = \dfrac{e}{Z_a + \dfrac{1}{j\omega C}} \quad \text{or} \quad \dfrac{je\omega C}{1 + jZ_a\omega C}$ ... (1)

and $$V = i \cdot \frac{1}{j\omega C}.\qquad \text{... (2)}$$

On substituting in equation (2) the value of i obtained in equation (1) we get

$$V = \frac{e}{1 + jZ_a\omega C}.\qquad \text{... (3)}$$

But, by definition, $\quad$ magnification $= \dfrac{V}{e}$

$$\therefore \frac{V}{e} = \frac{1}{1 + jZ_a\omega C} = \frac{1}{1 + j\omega C(R_a + j\omega L_a)} = \frac{1}{(1 - \omega^2 L_a C) + j\omega R_a C}.\qquad \text{... (4)}$$

At resonance V is a maximum, and then, e being constant, $\dfrac{V}{e}$ is a maximum too.

But $$\left(\frac{V}{e}\right)^2 = \frac{1}{(1 - \omega^2 L_a C)^2 + (\omega R_a C)^2}.\qquad \text{... (5)}$$

Let $$D = (1 - \omega^2 L_a C)^2 + (\omega R_a C)^2.\qquad \text{... (6)}$$

Then $\left(\dfrac{V}{e}\right)^2$ is a maximum when $\dfrac{V}{e}$ is a maximum, and when $\left(\dfrac{V}{e}\right)^2$ is a maximum, D should be a minimum.

The condition for D to be a minimum is $\dfrac{d(D)}{dC} = 0$.

On differentiating expression (6), and equating to zero, we get

$$2(1 - \omega^2 L_a C)(-\omega^2 L_a) + 2\omega^2 C R_a^2 = 0.\qquad \text{... (7)}$$

This expression, on simplification, gives

$$C = \frac{L_a}{Z_a^2}.\qquad \text{... (8)}$$

Substituting this value of C in equation (5) and simplifying results:

$$\left(\frac{V}{e}\right)^2 = \left(\frac{Z_a}{R_a}\right)^2 \quad \text{or} \quad \frac{V}{e} = \frac{Z_a}{R_a}, \quad i.e., \ \frac{V}{e} = \frac{R_a + j\omega L_a}{R_a}\qquad \text{... (9)}$$

i.e., $$\frac{V}{e} = 1 + j\frac{\omega L_a}{R_a}.\qquad \text{... (10)}$$

Thus the valve-voltmeter deflection is proportional to

$$\sqrt{1 + Q^2} \doteq Q.$$

Hence the meter dial can be so calibrated that it gives $\frac{\omega L_a}{R_a}$, which is the apparent Q of the coil.

CORRECTIONS

As stated already, the 'Q' as read on the meter dial is not the true Q, but it is the apparent Q.

Foster and Newlon[1] state that the following corrections are required for the readings of a 'Q' meter above 9 Mc/s in the meter they worked with. These corrections will be discussed below as to how far they are really warranted.

The corrections are due to:

(1) the loading due to the valve-voltmeter,
(2) the internal impedance of the meter source,
(3) the self-capacity of the coil.

(1) This does not arise at all in the Marconi-Ekco or other similar meter, since the makers state that a special triode valve (Ac/HL/DD) having a very high input resistance is used.

(2) Internal impedance of the meter consists of two (principal) factors:

(*a*) R_m, the internal resistance

and (*b*) L_m, the internal inductance of the meter.

Then Q_t, the true Q of the coil is given by $\frac{\omega L_t}{R_t}$ (11)

and Q_a, the apparent Q of the coil by $\frac{\omega L_a}{R_a}$. (12)

But
$$L_t = L_a - L_m \tag{13}$$
and
$$R_t = R_a - R_m \tag{14}$$

$$\frac{Q_t}{Q_a} = \frac{\omega L_t}{R_t} \times \frac{R_a}{\omega L_a} = \frac{L_t}{L_a} \times \frac{R_a}{R_t}$$

or
$$Q_t = Q_a \frac{(L_a - L_m)}{L_a} \times \frac{R_a}{(R_a - R_m)}. \tag{15}$$

Evaluation of L_a *and* L_t :

L_a is obtained as follows from the meter readings.

Since at resonance
$$f = \frac{1}{2\pi\sqrt{L_a(C+C_0)}} \tag{16}$$

$$L_a = \frac{1}{4\pi^2 f^2 (C+C_0)}, \tag{17}$$

where f is the frequency of the oscillator as read on the frequency calibration scale and C, the condenser reading on the meter, and C_0, the self capacity, which can be evaluated either by the negative intercept method (from the graph of $\frac{1}{f^2}$ *vs.* C or otherwise.

If f is in megacycles, C and C_0 in $\mu\mu f$, L_a in μh is obtained by

$$\frac{25330}{f^2(C+C_0)} \tag{18}$$

and the true inductance L_t (in μh) $= \frac{25330}{f^2(C+C_0)} - L_m$. ... (19)

$L_m = 0.05\mu h$ in the Marconi-Ekco meter the author used, and may even be neglected, when L_a is comparatively large as that 'Q' meter can measure the inductance of coils in the range $5\mu h$ to $25 mh$ approximately with an accuracy of $\pm 3\%$:

Thus, if L_m is neglected, then equation (15) reduces to

$$Q_t = Q_a \frac{R_a}{R_a - R_m} \quad \ldots \quad (20)$$

The need for the correction factor $\frac{R_a}{R_a - R_m}$ will solely depend on the relative values of R_a and R_m. If $R_m = 0.04\Omega$ and R_a even of the order of a few ohms, then $\frac{R_a}{R_a - R_m} \doteqdot 1$, when this correction also can be ignored without appreciable error.

(3) Correction for self-capacity of the coil is by far the most important correction, which need be applied below $15 mc/s$.

An expression for this correction is derived as follows :

If f_0 is the natural frequency of the coil, f the frequency at which Q is measured then

$$f_0 = \frac{1}{2\pi\sqrt{LC_0}} \quad \text{and} \quad f = \frac{1}{2\pi\sqrt{LC}} \quad \ldots \quad (21)$$

$$\left.\begin{array}{ll} \text{Let } k^2 \text{ denote the ratio} & \frac{\omega^2}{\omega_0^2} = \left(\frac{f}{f_0}\right)^2 = \frac{C_0}{C} \\ \text{Further,} & \frac{1}{\omega_0^2} = LC_0 \\ \therefore & \frac{\omega^2}{\omega_0^2} = \omega^2 LC_0 = k^2. \end{array}\right\} \quad \ldots \quad (22)$$

It is well known that the effective inductance L_e and effective resistance R_e of a coil at a frequency $\omega = 2\pi f$ are approximately given by the following expressions[2] where L and R are the physical or absolute values of inductance and resistance of the coil, respectively. These apparent values at a frequency f are a consequence of the self-capacity C_0 of the coil.

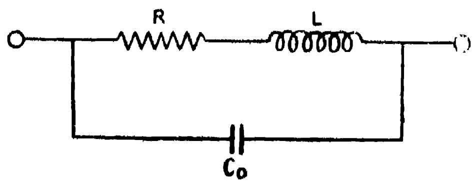

FIG. 3
Equivalent circuit of a coil at high frequencies

Fig. 3 shows the equivalent circuit at high frequencies of a coil having resistance and distributed or sellf-capacitance.

$$L_e \doteqdot \frac{L}{1-\omega^2 LC_0} \qquad (23)$$

and

$$R_e \doteqdot \frac{R}{(1-\omega^2 LC_0)^2} \qquad \dots \quad (24)$$

Substituting these values of L_e and R_e in equation (12) for L_a and R_a respectively, we get $Q_a = \frac{\omega L(1-k^2)^2}{R(1-k^2)} = \frac{\omega L\,(1-k^2)}{(R)}$,

i.e., $$Q_a = Q\,(1-k^2) \qquad \left(\because\ Q = \frac{\omega L}{R}\right)$$

$$\therefore \qquad \text{Actual } Q = \frac{Q_a}{1-k^2} \qquad \dots \quad (25)$$

But $k^2 = \frac{C_0}{C}$ as proved in equation (22).

$$\therefore \qquad Q = \frac{Q_a}{1-\frac{C_0}{C}} = Q_a\left\{\frac{C}{C-C_0}\right\} \text{ or } Q_a\left\{1-\frac{C_0}{C}\right\}^{-1} \qquad \dots \quad (26)$$

Expanding $\left\{1-\frac{C_0}{C}\right\}^{-1}$ and neglecting the terms beyond the second, we get

$Q = Q_a\left(1+\frac{C_0}{C}\right)$, which is the only important correction below 15 Mc/s ... (27)

ORDER OF APPLYING CORRECTIONS

When more than one correction is required, they should be made in the following order, after having ruled out the necessity for correction as a result of the valve-voltmeter loading :

(1) Internal impedance of the meter

and (2) Self-capacitance of the coil.

At each stage the semi-corrected values of Q and L should be used for the apparent value of $Q(Q_a)$ and $L(L_a)$ in succeeding corrections.

RADIO ENGINEER,
GOVERNMENT OF MADRAS.

REFERENCES

[1] Dudley E. Foster and Arthur E Newlon, "Measurement of Iron Cores at Radio Frequencies," *Proc. I.R.E.*, **29**, 269, 274-75 ; May (1941).

[2] Bur. Standards Circ. C. **74** : Radio Instruments and Measurements, 133 (1937).